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Asymptotic Behavior of Solutions to the

Damped Quasilinear Equation

$$\frac{\partial^2}{\partial t^2} u(x,t) + \gamma \frac{\partial u(x,t)}{\partial t} - \frac{\partial}{\partial x} \sigma\left(\frac{\partial u(x,t)}{\partial x}\right) = 0 \quad (1)$$

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Abstract

Asymptotic lower bounds for the L^2 norms of solutions of initial-boundary value problems associated with the equation of the title are derived for a simple case in which the equation fails to exhibit strict hyperbolicity. It is shown that in such cases it can be expected that the norm of a solution will be bounded away from zero as $t \to +\infty$ even as the damping factor γ becomes infinitely large.

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Initial boundary value problems associated with damped, first order quasilinear systems of the form

$$
\text{(s)} \qquad
\begin{aligned}
& w_t(x,t) - v_x(x,t) = 0 \\
& v_t(x,t) - \sigma(w(x,t))_x + \gamma v(x,t) = 0 \quad ,
\end{aligned}
$$

$\gamma > 0$, arise in several areas of nonlinear continuum mechanics and, in particluar, in the theory of shearing motions in nonlinear elastic solids in the presence of linear damping as well as in the theory of shearing perturbations of steady shearing flows in a nonlinear viscoelastic fluid; this latter case has recently been studied by Slemrod [1], [2], at least in those situations where the response of the fluid is such that (s) represents a strictly hyperbolic system, i.e. that $\sigma'(\zeta) \geq \epsilon > 0$, $\zeta \epsilon R^1$ (actually, the work in [1], [2] only requires for its validity that the nonlinearity σ satisfy $\sigma'(0) > 0$ and that the initial data $v(x,0)$, $w(x,0)$ be sufficiently small in an appropriate sense). By using a Riemann invariants argument Slemrod [1], [2] has been able to prove that in either of the situations delineated above smooth solutions (i.e., solutions which are of class C^1 in (x,t) jointly) must breakdown in finite time if the gradients of the initial data functions are sufficiently large in magnitude; his work thus compliments the earlier work of Nishida [3] who proved the global existence of smooth solutions to initial-boundary value problems associated with (s) under the assumptions that $\sigma'(0) > 0$ and that both the data functions and their gradients are sufficiently small in magnitude. The results in [1]-[3] no longer remain applicable if either $\sigma'(0) = 0$ or if $\sigma'(0) > 0$,

$\sigma'(\zeta) < 0$ for $|\zeta|$ sufficiently large, but the initial data are not chosen sufficiently small to guarantee that $\sigma'(w(x,t)) > 0$ for as long as smooth solutions of (s) exist; such cases would arise, for example, in the theory of shearing perturbations of steady flows in a nonlinear viscoelastic fluid if the fluid is of grade three, i.e. $\sigma(\zeta) = \sigma_1\zeta + \sigma_3\zeta^3$, and the material response is such that either $\sigma_1 = 0$, $\sigma_3 \neq 0$ or $\sigma_1 > 0$ but $\sigma_3 < 0$.

It is well known that (at least in a simply connected domain of (x,t) space) the system (s) is equivalent to (set $w = u_x$, $v = u_t$) the damped, quasilinear equation

$$\text{(e)} \quad u_{tt}(x,t) + \gamma u_t(x,t) - \sigma(u_x(x,t))_x = 0$$

and that if (v,w) is a sufficiently smooth solution of (s) then $w(x,t)$ satisfies

$$(\tilde{e}) \quad w_{tt}(x,t) + \gamma w_t(x,t) - \sigma(w(x,t))_{xx} = 0$$

By working with $(\tilde{e})$ we have managed [4] to show that, under appropriate hypotheses on the initial data, smooth solutions of associated initial-boundary value problems can not exist globally in time in the cases $\sigma'(0) = 0$ or $\sigma'(\zeta) < 0$ for $|\zeta|$ sufficiently large; by a smooth solution of $(\tilde{e})$ in [4] we mean, for example (in the case of associated homogeneous boundary data $w(0,t) = w(1,t) = 0$, $t > 0$) a function $w \in C^2((0,1)\times[0,\infty))$ such that $w_x^2(0,\cdot) \in L^1(0,\infty) \cap L^\infty(0,\infty)$, with analogous definitions in the case of either Neumann or mixed boundary conditions.

Decay to zero in the L^{∞} norm, as $t \to +\infty$, for the unique smooth globally defined solution of initial boundary-value problems associated with (e), in the strictly hyperbolic situation, has been established by Nishida in [3] by using a variant of the L^2 - energy method of Courant - Friedrichs - Lewy [5]. (Similiar arguments have been employed recently by Dafermos and Nohel [6], [7] to treat the asymptotic stability of solutions to some non-linear integrodifferential equations arising in theories of nonlinear visco-elastic response, which differ from the theory employed in [1], [2], and by Slemrod [8] to prove the asymptotic stability of solutions to a system of quasilinear equations associated with nonlinear thermoelastic response).

As with the global existence and nonexistence theorems in [1]-[3] the asymptotic stability results in [3], and the method used to establish them, fail to apply in those situations where either $\sigma'(0) = 0$ or $\sigma'(\zeta) < 0$ for $|\zeta|$ sufficiently large (i.e., for $|\zeta|$ sufficiently large, hyperbolicity breaks down and (e) becomes, in essence, a quasilinear elliptic equation). For linear elliptic equations of the form

$$(\hat{e}) \quad u_{tt} + \gamma u_t + c u_{xx} = 0 \; ; \quad \gamma > 0 \; , \; c > 0$$

it follows from abstract results of this author [9] that it is possible to choose $u(x,0)$ so large that as $t \to +\infty$ the L_2 norm of u on a finite interval, say $[0,1]$, will be bounded away from zero even as the damping factor $\gamma \to +\infty$. To be more precise, it follows from the results of [9] that for solutions of the initial-boundary value problem

$$
(1.1)\qquad
\begin{aligned}
& u^{\alpha}_{tt} + \gamma u^{\alpha}_{t} + c u^{\alpha}_{xx} = 0 \quad , \quad 0 \le x \le 1 \ , \ t < 0 \\
& u^{\alpha}(0,t) = 0 \quad , \ u^{\alpha}(1,t) = 0 \quad , \ t > 0 \\
& u^{\alpha}(x,0) = \alpha\, \bar{u}(x) \ , \ u^{\alpha}_{t}(x,0) = \bar{v}(x) \ , \ 0 \le x \le 1
\end{aligned}
$$

it is true that

$$
(1.2)\qquad
\begin{aligned}
& \lim_{\gamma\to+\infty} \ \lim_{t\to+\infty} \ \|u^{\alpha}(\cdot,t)\|^2_{L^2(0,1)} \ge \alpha^2 \ \|\bar{u}(\cdot)\|^2_{L^2(0,1)} \\
& \lim_{t\to\infty} \ \lim_{\gamma\to+\infty} \ \|u^{\alpha}(\cdot,t)\|_{L^2(0,1)} \ge \alpha^2 \ \|\overline{u(\cdot)}\|^2_{L^2(0,1)}
\end{aligned}
$$

provided only that $\|\bar{u}\|_{H^1(0,1)} > 0$ and that α is chosen so as to satisfy

$$
(1.3)\qquad \alpha > \|\bar{v}\|_{L^2(0,1)} / c\ \|\bar{u}\|^2_{H^1(0,1)}
$$

It is assumed, of course, that $\bar{u}(\cdot), \bar{v}(\cdot) \in H^1_0(0,1)$.

It is the purpose of this note to prove, using entirely elementary arguments, that solutions of (e) must behave, as $t\to+\infty$, in a manner analogous to those of $(\hat{e})$ *when we do not assume strict hyperbolicity.* Our results cover simple situations in which $\sigma'(\zeta) \le 0$, $\forall \zeta \in R^1$,so that (e) models an essentially elliptic situation, but we conjecture that similar results hold in the more delicate situation where $\sigma'(0) > 0$ but $\sigma'(\zeta) < 0$, for $|\zeta|$ sufficiently large, with the initial data not chosen so small so as to gurantee that (e) remains hyperbolic for as long as sufficiently smooth solutions exist. To this end, consider (e) with $u(x,t)$ replaced by $u^{\alpha}(x,t)$ and associated initial and boundary data of the type present in (1.1), i.e., consider the system

$$(1.1^*)\qquad \begin{aligned} &u^\alpha_{tt} + \gamma u^\alpha_t - \sigma(u^\alpha_x)_x = 0\ ,\ 0 \le x \le 1\ ,\ t > 0 \\ &u^\alpha(0,t) = 0\ ,\ u^\alpha(1,t) = 0 \\ &u^\alpha(x,0) = \alpha\bar{u}\,(x)\ ,\ u^\alpha_t(x,0) = \bar{v}(x)\ ;\ \ 0 \le x \le 1 \end{aligned}$$

Instead of the Dirichlet conditions in (1.1*) we could work equally well with Neumann type boundary conditions $u^\alpha_x(0,t) = u^\alpha_x(1,t) = 0$, $t > 0$, if $\sigma(\zeta)$ satisfies $\sigma(0) = 0$, in addition to hypothesis (σ) below; in (1.1*) $\gamma > 0$, $\alpha > 0$ and we assume only that $\bar{u}(\cdot)$, $\bar{v}(\cdot) \in H^1_o(0,1)$ (for the Dirichlet conditions) and $\bar{u}(\cdot)$, $\bar{v}(\cdot) \in H^1(0,1)$ with $\bar{u}_x(\cdot)$, $\bar{v}_x(\cdot) \in H^1_o(0,1)$ for the Neumann conditions. In both situations we assume that $\|\bar{u}(\cdot)\|_{L^2(0,1)} > 0$ and that $\langle\bar{u}(\cdot),\ \bar{v}(\cdot)\rangle_{L^2(0,1)} \neq 0$. Concerning the nonlinearity $\sigma(\cdot)$ we assume that $\sigma: R^1 \to R^1$ with $\sigma \in C^1(R^1)$ and

$$(\sigma)\qquad \zeta\sigma(\zeta) \le 0\ ,\quad \text{for all } \zeta\epsilon R^1\ .$$

This hypothesis is satisfied, for example, for $\sigma(\zeta) = \sigma_3\zeta^3$ with $\sigma_3 < 0$ in which case $\sigma'(0) = 0$, $\sigma'(\zeta) \le 0$, $\forall\,\zeta\epsilon R^1$ and (c) becomes

$$(1.4.)\qquad u_{tt} + \gamma u_t + 3|\sigma_3|u^2_x\, u_{xx} = 0$$

Now, let $H_\alpha(t) = H(u^\alpha(\cdot,t)) = \|u^\alpha(\cdot,t)\|^2_{L^2(0,1)}$, $t > 0$, which is well-defined on solutions $u^\alpha(\cdot,t)\epsilon L^2(0,1)$ of (1.1*) for all $t > 0$. Clearly

$$\begin{aligned} H'_\alpha(t) &= 2\ \langle u^\alpha_t(\cdot,t),\ u^\alpha(\cdot,t)\rangle_{L^2(0,1)} \quad \text{and} \\ H''_\alpha(t) &= 2\|u^\alpha_t(\cdot,t)\|^2_{L^2(0,1)} + 2\ \langle u^\alpha_{tt}(\cdot,t),\ u^\alpha(\cdot,t)\rangle_{L^2(0,1)} \\ &= 2\|u^\alpha_t(\cdot,t)\|^2_{L^2(0,1)} - 2\gamma\ \langle u^\alpha_t(\cdot,t),\ u^\alpha(\cdot,t)\rangle_{L^2(0,1)} \\ &\quad + 2\ \langle u^\alpha(\cdot,t),\ -(u^\alpha_x(\cdot,t))_x\rangle_{L^2(0,1)} \end{aligned}$$

in view of (1.1*). Using the expression for $H'_\alpha(t)$ we then have

$$(1.5)\qquad H''_\alpha(t) + \gamma H'_\alpha(t) = 2\langle u^\alpha(\cdot,t),\ \sigma(u^\alpha_x(\cdot,t))_x\rangle_{L^2(0,1)} + 2\|u^\alpha_t(\cdot,t)\|^2_{L^2(0,1)}$$

But,

$$\begin{aligned}\langle u^\alpha(\cdot,t),\ \sigma(u^\alpha_x(\cdot,t))_x\rangle &= \int_0^1 u^\alpha(x,t)\sigma(u^\alpha_x(x,t))_x dx \\ &= u^\alpha(x,t)\sigma(u^\alpha_x(x,t))\Big|_0^1 \\ &\quad - \int_0^1 u^\alpha_x(x,t)\sigma(u^\alpha_x(x,t))dx \\ &= -\int_0^1 u^\alpha_x(x,t)\sigma(u^\alpha_x(x,t))dx \\ &\geq 0 \quad , \quad t > 0\end{aligned}$$

in view of the boundary conditions and our hypothesis (σ). [If we are working with the Neumann conditions then $\sigma(0) = 0$ yields immediately that $\sigma(u^\alpha_x(0,t)) = \sigma(u^\alpha_x(1,t)) = 0 \quad , \quad t > 0$]. Thus, by (1.5) we have

$$(1.6)\qquad H''_\alpha(t) \geq -\gamma H'_\alpha(t) \quad , \ t > 0$$

One integration of this equation yields

$$\frac{d}{dt}(e^{\gamma t}H_\alpha(t)) \geq e^{\gamma t}(H'_\alpha(0) + \gamma H_\alpha(0))$$

and a second integration then produces the estimate

$$\begin{aligned}(1.7)\qquad H_\alpha(t) &\geq e^{-\gamma t}H_\alpha(0) + \frac{(1-e^{-\gamma t})}{\gamma}(H'_\alpha(0) + \gamma H_\alpha(0)) \\ &= H_\alpha(0) + \frac{(1-e^{-\gamma t})}{\gamma} H'_\alpha(0)\end{aligned}$$

or, if we reintroduce the definition of $H_\alpha(t)$ and rewrite $H_\alpha(0)$, $H'_\alpha(0)$ in terms of the initial data

$$(1.8) \qquad \|u^\alpha(\cdot,t)\|^2_{L^2(0,1)} \geq \alpha^2 \|\bar{u}(\cdot)\|^2_{L^2(0,1)} + \frac{2\alpha}{\gamma}\langle \bar{u}(\cdot), \bar{v}(\cdot)\rangle_{L^2(0,1)}(1-e^{-\gamma t})$$

Clearly, if $\langle \bar{u}(\cdot), \bar{v}(\cdot)\rangle_{L^2(0,1)} > 0$ then it follows from (1.8) that for any fixed $\alpha > 0$, $\gamma > 0$

$$(1.9) \qquad \lim_{t\to\infty} \|u^\alpha(\cdot,t)\|^2_{L^2(0,1)} \geq \alpha^2 \|\bar{u}(\cdot)\|^2 + \frac{2\alpha}{\gamma}\langle \bar{u}(\cdot), \bar{v}(\cdot)\rangle_{L^2(0,1)}$$

$$\equiv \mathcal{J}(\alpha, \gamma; \bar{u}, \bar{v}) > 0$$

and that for any fixed $\alpha > 0$

$$(1.10) \qquad \begin{aligned} &\lim_{t\to\infty} \lim_{\gamma\to\infty} \|u^\alpha(\cdot,t)\|^2_{L^2(0,1)} \geq \alpha^2 \|\bar{u}(\cdot)\|^2_{L^2(0,1)} \\ &\lim_{\gamma\to\infty} \lim_{t\to\infty} \|u^\alpha(\cdot,t)\|_{L^2(0,1)} \geq \alpha^2 \|\bar{u}(\cdot)\|^2_{L^2(0,1)} \end{aligned}$$

On the other hand, if $\langle \bar{u}(\cdot), \bar{v}(\cdot)\rangle_{L^2(0,1)} < 0$ then from (1.8) we obtain

$$(1.11) \qquad \lim_{t\to\infty} \|u^\alpha(\cdot,t)\|^2_{L^2(0,1)} \geq \alpha^2 \|\bar{u}(\cdot)\|^2_{L^2(0,1)} - \frac{2\alpha}{\gamma} \left|\langle \bar{u}(\cdot), \bar{v}(\cdot)\rangle_{L^2(0,1)}\right|$$

$$\equiv \mathcal{J}(\alpha, \gamma; \bar{u}, \bar{v}) > 0$$

provided we choose

$$\alpha = \alpha_\gamma > \left(\frac{2}{\gamma}\right) \frac{\left|\langle \bar{u}(\cdot), \bar{v}(\cdot)\rangle_{L^2(0,1)}\right|}{\|\bar{u}(\cdot)\|^2_{L^2(0,1)}}$$

In this case it follows that for fixed $\gamma \in (0,\infty)$ we may choose $\alpha = \alpha_\gamma$

so large that $\|u^{\alpha_\gamma}(\cdot,t)\|^2_{L^2(0,1)}$ is bounded away from zero as $t \to +\infty$.

Clearly $\alpha_\gamma \to 0^+$ as $\gamma \to +\infty$. On the other hand for arbitrary $\alpha > 0$ it follows at once from (1.11) that the limits in (1.10) are valid even when $\langle \bar{u}(\cdot), \bar{v}(\cdot)\rangle_{L^2(0,1)} < 0$.

Before summarizing the above results in a formal theorem it is worth noting that slightly sharper estimates can be obtained with only a little more work. In order to obtain such estimates we begin by computing directly that for any $\beta > 0$

$$(1.12) \qquad H_\alpha(t)H''_\alpha(t) - (\beta+1)H'^2_\alpha(t) \geq 2H_\alpha(t)\mathscr{S}_{\alpha,\beta}(t)$$

where

$$(1.13) \qquad \mathscr{S}_{\alpha,\beta}(t) = \langle u^\alpha(\cdot,t), u^\alpha_{tt}(\cdot,t)\rangle_{L^2(0,1)} - (2\beta+1)\|u^\alpha_t(\cdot,t)\|^2_{L^2(0,1)}$$

The estimate (1.12) depends only on the form of H_α and is independent of the particular equation satisfied by $u^\alpha(\cdot,t)$ (e.g., see Levine [10]) Substituting in (1.13) from (1.1*) we then obtain

$$(1.14) \qquad \mathscr{S}_{\alpha,\beta}(t) = \langle u^\alpha(\cdot,t), \sigma(u^\alpha_x(\cdot,t))_x\rangle_{L^2(0,1)} - \frac{\gamma}{2}H'_\alpha(t) - (2\beta+1)\|u^\alpha_t(\cdot,t)\|^2_{L^2(0,1)}$$

However, by (1.5) it follows that

$$\|u_t^\alpha(\cdot,t)\|^2_{L^2(0,1)} = \frac{1}{2} H''_\alpha(t) + \frac{\gamma}{2} H'_\alpha(t)$$
$$- \langle u^\alpha(\cdot,t), \sigma(u_x^\alpha(\cdot,t))_x \rangle_{L^2(0,1)}$$

and, therefore, as $\langle u^\alpha(\cdot,t), \sigma(u_x^\alpha(\cdot,t))_x \rangle_{L^2(0,1)} \geq 0$, $\forall\, t > 0$,

by virtue of hypothesis (c) and the boundary conditions, it follows that

$$(1.15) \qquad -(2\beta+1)\|u_t^\alpha(\cdot,t)\|^2_{L^2(0,1)} \geq -\left(\frac{2\beta+1}{2}\right)H''_\alpha(t)$$
$$- \gamma\left(\frac{2\beta+1}{2}\right)H'_\alpha(t)$$

Introducing the estimate (1.15) into (1.14) we obtain as a lower bound for $\mathcal{J}_{\alpha,\beta}(t)$

$$(1.16) \qquad \mathcal{J}_{\alpha,\beta}(t) \geq \langle u^\alpha(\cdot,t), \sigma(u_x^\alpha(\cdot,t))_x \rangle_{L^2(0,1)}$$
$$- \gamma(\beta+1)H'_\alpha(t) - \left(\frac{2\beta+1}{2}\right) H''_\alpha(t)$$

We now substitute from (1.16) into (1.12) (after first dropping the nonnegative term $\langle u^\alpha(\cdot,t), \sigma(u_x^\alpha(\cdot,t))_x \rangle_{L^2(0,1)}$) and then rearrange terms and divide through by $(\beta+1)$ so as to obtain the differential inequality

$$(1.17) \qquad H_\alpha(t)H''_\alpha(t) - \frac{1}{2}\; H'^2_\alpha(t) \geq -\gamma\, H_\alpha(t)H'_\alpha(t)$$

A simple computation shows that (1.17) is equivalent to

$$(1.18) \qquad [H^{1/2}_\alpha(t)]'' \geq -\gamma\, [H^{1/2}_\alpha(t)]'$$

A first integration of (1.18) then yields the estimate

$$\frac{d}{dt}(e^{\gamma t}H_{\alpha}^{1/2}(t)) \geq e^{\gamma t}(H_{\alpha}^{1/2'}(0) + \gamma H_{\alpha}^{1/2}(0))$$

while a second integration yields

$$H_{\alpha}^{1/2}(t) \geq H_{\alpha}^{1/2}(0) + \frac{(1-e^{-\gamma t})}{\gamma}H_{\alpha}^{1/2'}(0) \tag{1.19}$$

Rewriting (1.19) using the definition of $H_{\alpha}(t)$ we easily obtain the estimate

$$\begin{aligned} \|u^{\alpha}(\cdot,t)\|_{L^2(0,1)} &\geq \alpha\|\bar{u}(\cdot)\|_{L^2(0,1)} \\ &\quad + \left(\frac{1}{\gamma}\right)\frac{\langle\bar{u}(\cdot),\ \bar{v}(\cdot)\rangle_{L^2(0,1)}}{\|\bar{u}(\cdot)\|_{L^2(0,1)}}(1-e^{-\gamma t}) \end{aligned} \tag{1.20}$$

from which, for arbitrary $\alpha > 0$ and either $\langle\bar{u}(\cdot),\ \bar{v}(\cdot)\rangle_{L^2(0,1)} > 0$ or $\langle\bar{u}(\cdot),\ \bar{v}(\cdot)\rangle_{L^2(0,1)} < 0$ we get the obvious counter parts of (1.10). Also from (1.20) we find that for $\langle\bar{u}(\cdot),\ \bar{v}(\cdot)\rangle_{L^2(0,1)} > 0$ and $\alpha > 0$ arbitrary

$$\begin{aligned} \lim_{t\to\infty}\|u^{\alpha}(\cdot,t)\|_{L^2(0,1)} &\geq \alpha\|\bar{u}(\cdot)\|_{L^2(0,1)} \\ &\quad + \left(\frac{1}{\gamma}\right)\frac{\langle\bar{u}(\cdot),\ \bar{v}(\cdot)\rangle_{L^2(0,1)}}{\|\bar{u}(\cdot)\|_{L^2(0,1)}} \\ &\equiv \mathcal{K}(\alpha,\ \gamma;\ \bar{u},\ \bar{v}) > 0 \end{aligned} \tag{121}$$

while for $\langle\bar{u}(\cdot),\ \bar{v}(\cdot)\rangle_{L^2(0,1)} < 0$ and

$$\alpha = \bar{\alpha}_\gamma > (\frac{1}{\gamma}) \frac{|\langle \bar{u}(.),\ \bar{v}(.)\rangle_{L^2(0,1)}|}{\|\bar{u}(.)\|^2_{L^2(0,1)}}$$

$$(1.22) \qquad \lim_{t\to\infty} \|u^{\alpha_\gamma}(.,t)\|_{L^2(0,1)} \geq \alpha_\gamma \|\bar{u}(.)\|_{L^2(0,1)} - (\frac{1}{\gamma}) \frac{|\langle \bar{u}(.),\ \bar{v}(.)\rangle_{L^2(0,1)}|}{\|u(.)\|^2_{L^2(0,1)}}$$

$$\equiv \mathcal{L}(\bar{\alpha}_\gamma\ ,\ \gamma;\ \bar{u}'\bar{v}) > 0$$

for any $\gamma \epsilon (0,\infty)$. In other words for α_γ sufficiently large $\|u^{\alpha_\gamma}(.,t)\|_{L^2(0,1)}$ is bounded away from zero as $t\to+\infty$. We summarize our results in the following

<u>Theorem</u> Let $u^\alpha(x,t)$ denote a classical solution of (1.1*) where $\alpha > 0$, $\gamma > 0$ and assume that $\sigma: R^1 \to R^1$ is of class C^1 and satisfies (σ) . Then for arbitrary α , γ and arbitrary data $\bar{u}(.)$, $\bar{v}(.)$ in $H_0^1(0,1)$ $\|u^\alpha(.,t)\|^2_{L^2(0,1)}$ (respectively. $\|u^\alpha(.,t)\|_{L^2(0,1)}$) satisfies the growth estimate (1.8) (respectively, (1.20).). It thus follows that for data $\bar{u}(.)$, $\bar{v}(.)$ such that $\langle \bar{u}(.)\ ,\ \bar{v}(.)\rangle > 0$ the estimate (1.9) (respectively, (1.21)) holds for any $\alpha > 0$ as $t\to+\infty$ while for $\langle \bar{u}(.)\ ,\ \bar{v}(.)\rangle < 0$ and fixed $\gamma \epsilon (0,\infty)$ it is possible to choose $\alpha = \alpha_\gamma$ so large that $\|u^{\alpha_\gamma}(.,t)\|^2_{L^2(0,1)}$ (respectively, $\|u^{\alpha_\gamma}(.,t)\|_{L^2(0,1)}$) satisfies (1.11) (respectively, (1.22))

as $t \to +\infty$. As long as $\langle \bar{u}(\cdot) , \bar{v}(\cdot) \rangle \neq 0$, $\|u^\alpha(\cdot,t)\|^2_{L^2(0,1)}$ satisfies (1.10) as both $\gamma \to \infty$, $t \to \infty$ for any $\alpha > 0$ while $\|u^\alpha(\cdot,t)\|_{L^2(0,1)}$ satisfies the obvious analogous results, for arbitrary $\alpha > 0$ as both $\gamma \to \infty$, $t \to \infty$. Similar results hold if $u_x^\alpha(0,t) = u_x^\alpha(1,t) = 0$ and $\sigma(0) = 0$.

There remains open the more interesting situation where, for example, $\sigma(\zeta) = \sigma_1\zeta + \sigma_3\zeta^3$ with $\sigma_1 > 0$, $\sigma_3 < 0$ so that $\sigma'(\zeta) < 0$ for $|\zeta|$ sufficiently large. In this case (σ) is satisfied not for all $\zeta \in R^1$ but only for $\zeta \in R^1$ with $|\zeta|$ sufficiently. While we conjecture that asymptotic lower bounds of the type described in the above Theorem still hold in this situation as well we have not yet been able to produce a proof. A more difficult problem would seem to be to find the most general hypotheses relative to $\sigma(\zeta)$ which would imply the kind of asymptotic behavior described in the Theorem.

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